RUTGERS
Rutgers Business School

26:711:563: Stochastic Calculus for Finance Spring 2018 1 Washington Park, room 118 [Newark] Tuesday 10—12:50	**Dr. Andrzej Ruszczynski** 100 Rockefeller Rd, room 5182 (848) 445-3184 rusz@business.rutgers.edu office hours: Tue 3-5 PM or by appt.

COURSE DESCRIPTION

The course will provide the students with rigorous introduction to the theory of stochastic calculus and its applications in finance. It will start from random sequences and analysis of different convergence concepts. Discrete-time martingales will be introduced and several important martingale inequalities proved. Discrete-time random walk will be introduced and analyzed. Several results, including the reflection principle will be presented. These concepts will be used to construct and analyze a discrete-time model of the evolution of stock prices. The concept of arbitrage will be introduced in discrete time, and arbitrage-free valuation of contingent claims will be derived by martingale methods. Next, the Brownian motion process will be introduced and analyzed. The reflection principle will be used to derive important properties of the Brownian motion process. The concept of a continuous-time martingale will be introduced, and several properties of martingales proved. The concept of the stochastic integral will be introduced. The quadratic variation process will be defined. Stochastic differential equations will be discussed. Ito formula, integration by parts, and change of the order of integration will be presented. Integration with respect to a martingale will be covered as well. Further topics include the Girsanov theorem, Brownian martingale representation, and the Feynman-Kac equation. Geometric Brownian motion will be used to construct a continuous-time model of the evolution of a stock price, and all mathematical concepts and results will be illustrated with applications to finance. In particular, the Black-Scholes option pricing theory will be developed and the Feynman-Kac partial differential equation will be specialized to finance. Applications to complex contingent claims will be presented. Stopping problems and American options will be covered, together with basic numerical methods for their valuation. Finally, multifactor models will be discussed. The course will finish with an overview of stochastic volatility models.

COURSE MATERIALS

Textbook: [Required]. A. Etheridge, *A Course in Financial Calculus*, Cambridge University Press 2002, ISBN-13: 978-0521813853
Textbook: [Supplementary]. E. Cinlar, *Probability and Stochastics*, Springer, 2011, ISBN-13: 978-0387878584
Textbook: [Supplementary].   I. Karatzas and S. E. Shreve, *Brownian Motion and Stochastic Calculus*, Springer, 1997, ISBN-13: 978-0387965352
Textbook: [Supplementary].   M. Musiela and M. Rutkowski, *Martingale Methods in Financial Modelling*, Springer, 1997, ISBN-13: 978-3540209669

- Check Blackboard (blackboard.rutgers.edu) and your official Rutgers email account regularly.

LEARNING GOALS AND OBJECTIVES

- This course is designed to help students develop skills and knowledge in the following area:

 Quantitative Finance knowledge.

The students will have a command of stochastic calculus theory and its application to mathematical and computer modeling of securities prices, derivatives, and interest rates.

- Students who complete this course will demonstrate the following:

(a) Mastery in designing lattice stock models, models by stochastic differential equations, theoretical and numerical methods of pricing derivatives.
(b) Ability to integrate and apply stochastic analysis tools to solve practical financial engineering problems.

- Students will develop these skills and knowledge through the following course activities and assignments:

Assignments 1 and 2 will help develop skills to build and analyze lattice models. Assignments 3 through 6 will help understand stock price models by stochastic differential equations. Assignments 7 through 9 are devoted to martingale pricing of tradeable assets, in particular, derivatives. Assignment 10 focuses on pricing of American options and dynamic programming, while Assignment 11 covers stochastic volatility models.

PREQUISITES

There are no formal prerequisites, but good preparation in multivariate calculus and the theory of probability is required.

ACADEMIC INTEGRITY

I do NOT *tolerate cheating*. Students are responsible for understanding the RU Academic Integrity Policy (https://slwordpress.rutgers.edu/academicintegrity/wp-content/uploads/sites/41/2014/11/AI_Policy_2013.pdf). I will strongly enforce this Policy and pursue *all* violations. On all examinations and assignments, students must sign the RU Honor Pledge, which states, "On my honor, I have neither received nor given any unauthorized assistance on this examination or assignment." [I will screen all written assignments through *SafeAssign* or *Turnitin*, plagiarism detection services that compare the work against a large database of past work.] Don't let cheating destroy your hard-earned opportunity to learn. See business.rutgers.edu/ai for more details.

ATTENDANCE AND PREPARATION POLICY

- Expect me to attend all class sessions. I expect the same of you. If I am to be absent, my department chair or I will send you notice via email and Blackboard as far in advance as possible. If you are to be absent, report your absence in advance at https://sims.rutgers.edu/ssra/. If your absence is due to religious observance, a Rutgers-approved activity, illness, or family emergency/death and you seek makeup work, also send me an email with full details and supporting documentation [within 3 days of your first absence]. [Explain other aspects of your absence policy in detail; it will save you trouble later.]

- For weather emergencies, consult the campus home page. If the campus is open, class will be held.

- Expect me to arrive on time for each class session. I expect the same of you.

- Expect me to remain for the entirety of each class session. I expect the same of you.

- Expect me to prepare properly for each class session. I expect the same of you. Complete all background reading and assignments. You cannot learn if you are not prepared. The minimum expectation is that for each 3-hour class session, you have prepared by studying for at least twice as many hours.

- Expect me to participate fully in each class session. I expect the same of you. Stay focused and involved. You cannot learn if you are not paying attention.

CLASSROOM CONDUCT

The use of cell phones is forbidden. Please, refrain from eating and drinking. Do not sleep.

EXAM DATES AND POLICIES

There are 2 exams in this course:
Midterm Exam: Feb. 27, 2018. Open book exam.
Final Exam: May 1, 2018. Open book exam, comprehensive.

During exams, the following rules apply:
  - If you have a disability that influences testing procedures, provide me an official letter from the Office of Disability Services at the start of the semester.
  - No cell phones or other communication devices are allowed in the testing room.
  - You must show a valid Rutgers photo ID to enter the room and to turn in the exam.
  - Alternate seating; do not sit next to another student or in your usual seat.
  - Use the bathroom prior to the exam start; bathroom breaks, if essential, will be escorted.
  - Your exam will not be accepted unless you sign the Honor Pledge.

GRADING POLICY

Course grades will be determined by weighted average of the following components:
Homework assignments (30%), Midterm exam (30%), and Final exam (40%). A score above 90% will be graded as A, above 80% as B+, above 70% as B, above 60% as C+, and above 50% as C. Scores below 50% are failing grades.

In the case of an excellent score on the final exam I may raise the grade above the level provided in the table above.

Your final grade is not subject to negotiation. If you feel I have made an error, submit your written argument to me within one week of receiving your final grade. Clarify the precise error I made and provide all due supporting documentation. If I have made an error, I will gladly correct it. But I will adjust grades only if I have made an error. I cannot and will not adjust grades based on consequences, such as hurt pride, lost scholarships, lost tuition reimbursement, lost job opportunities, or dismissals. Do not ask me to do so. It is dishonest to attempt to influence faculty in an effort to obtain a grade that you did not earn, and it will not work.

COURSE SCHEDULE

Date	Topic	Items Due
Jan. 16	Review of probability theory	
Jan. 23	Discrete-time martingales	Assignment 1
Jan. 30	Binomial stock models. Valuation of claims	Assignment 2
Feb. 6	The Brownian motion	Assignment 3
Feb. 13	Continuous-time martingales	Assignment 4
Feb. 20	Introduction to stochastic integration	Assignment 5
Feb. 27	Midterm exam	
Mar. 6	Stochastic integral. Ito formula	
Mar. 20	Stochastic differential equations	Assignment 6
Mar. 27	Integration by parts. Girsanov theorem	Assignment 7
Apr. 3	The Feynman-Kac equation	Assignment 8
Apr. 10	The Black-Scholes Model	Assignment 9
Apr. 17	American options	Assignment 10
Apr. 23	Multifactor models. Stochastic volatility models.	Assignment 11
May 1	Final exam	

SUPPORT SERVICES

If you need accommodation for a *disability*, obtain a Letter of Accommodation from the Office of Disability Services. The Office of Disability Services at Rutgers, The State University of New Jersey, provides student-centered and student-inclusive programming in compliance with the Americans with Disabilities Act of 1990, the Americans with Disabilities Act Amendments of 2008, Section 504 of the Rehabilitation Act of 1973, Section 508 of the Rehabilitation Act of 1998, and the New Jersey Law Against Discrimination. https://ods.rutgers.edu

If you are a military *veteran* or are on active military duty, you can obtain support through the Office of Veteran and Military Programs and Services. http://veterans.rutgers.edu/

If you are in need of *mental health* services, please use our readily available services.
        [Select for inclusion in syllabus based on course location]
[Rutgers University-Newark Counseling Center: http://counseling.newark.rutgers.edu/]
[Rutgers Counseling and Psychological Services – New Brunswick: http://rhscaps.rutgers.edu/]

If you are in need of *physical health* services, please use our readily available services.
        [Select for inclusion in syllabus based on course location]
[Rutgers Health Services – Newark: http://health.newark.rutgers.edu/]
[Rutgers Health Services – New Brunswick: http://health.rutgers.edu/]

If you are in need of *legal* services, please use our readily available services: http://rusls.rutgers.edu/

If you are in need of additional *academic assistance*, please use our readily available services.
        [Select for inclusion in syllabus based on course location; undergraduate only]
[Rutgers University-Newark Learning Center: http://www.ncas.rutgers.edu/rlc
 Rutgers University-Newark Writing Center: http://www.ncas.rutgers.edu/writingcenter]
[Rutgers University-New Brunswick Learning Center: https://rlc.rutgers.edu/]